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On the Ternary Non-homogeneous Quintic Equation

$$x^2+3y^2=7z^5$$

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Abstract:

The ternary non-homogeneous quintic equation given by $x^2+3y^2=7z^5$ is analysed for determining its distinct integer solutions.Also,a generation formula for the integer solutions to the given quintic equation ,being given its particular solution, is illustrated.

Key words: Ternary quintic ,Non-homogeneous quintic , Integer solutions

Introduction:

It is well-known that the Diophantine equations ,homogeneous or non-homogeneous ,have aroused the interest of many mathematicians. In particular,one may refer [1-11] for quintic equations with three unknowns .The above problems motivated us to search for the distinct integer solutions to ternary non-homogeneous quintic equation $x^2+3y^2=7z^5$

Also,a general formula for generating sequence of integer solutions to the considered quintic equation being given its particular solution is illustrated.

Method of analysis:

The ternary non-homogeneous quintic equation under consideration is

$$x^2+3y^2=7z^5 \qquad (1)$$

The process of determining non-zero distinct integer solutions to (1) is illustrated below:

Method 1:

Assume

$$z = a^2 + 3b^2 \qquad (2)$$

Express the integer 7 on the R.H.S. of (1) as the product of complex conjugates as below

$$7 = (2 + i\sqrt{3})(2 - i\sqrt{3}) \qquad (3)$$

Using (2) & (3) in (1) and employing the method of factorization ,consider

$$x + i\sqrt{3}y = (a + i\sqrt{3}b)^5$$

Equating the rational and irrational parts , it is seen that

$$x = 2f(a,b) - 3g(a,b) \,, y = f(a,b) + 2g(a,b),$$
where
$$f(a,b) = a^5 - 30\,a^3 b^2 + 45\,a b^4 \,, g(a,b) = 5a^4 b - 30\,a^2 b^3 + 9b^5 \qquad (4)$$

Thus,(2) and (4) represent the integer solutions to (1).

Note:1

It is to be noted that ,apart from (3) ,the integer 7 on the R.H.S. of (1) is also represented as shown below:

$$7 = \frac{(5 + i\sqrt{3})(5 - i\sqrt{3})}{4} \qquad (5)$$

Following the analysis as above ,the corresponding integer solutions to (1) are given by

$$x = 2^4[5f(A,B) - 3g(A,B)] \,, y = 2^4[f(A,B) + 5g(A,B)] \,, z = 4(A^2 + 3B^2)$$

Method 2:

Rewrite (1) as

$$x^2 + 3y^2 = 7z^5 * 1 \qquad (6)$$

Consider the integer 1 on the R.H.S. of (6) as

$$1=\frac{(1+i\sqrt{3})(1-i\sqrt{3})}{4} \tag{7}$$

Substituting (2) ,(3) ,(7) in (6) and employing the method of factorization , consider

$$x+i\sqrt{3}y=(2+i\sqrt{3})\frac{(1+i\sqrt{3})}{2}[f(a,b)+i\sqrt{3}g(a,b)]$$

Following the analysis as in Method 1,the corresponding integer solutions to (1) are given by

$$x=2^4[-f(A,B)-9g(A,B)]\,,y=2^4[3f(A,B)-g(A,B)]\,,z=4(A^2+3B^2)$$

Note 2 :

Substituting (2) ,(5) ,(7) in (6) and repeating the above process ,it is seen that (1) is satisfied by

$$x=2^4[f(A,B)-9g(A,B)]\,,y=2^4[3f(A,B)+g(A,B)]\,,z=4(A^2+3B^2)$$

Note 3 :

In addition to (7) ,the integer 1 on the R.H.S. of (6) is expressed as

$$1=\frac{(3r^2-s^2+i\sqrt{3}2rs)(3r^2-s^2-i\sqrt{3}2rs)}{(3r^2+s^2)^2} \tag{8}$$

In this case ,the repetition of the above process on considering the equations (2) ,(3) ,(8) and equations (2) ,(5) ,(8) respectively leads to two more sets of integer solutions to (1).

Method 3:

Taking

$$x=(7k-5)y \tag{9}$$

in (1) , it simplifies to

$$(7k^2-10k+4)y^2=z^5$$

which is satisfied by

$$y=(7k^2-10k+4)^2\alpha^{5s},z=(7k^2-10k+4)\alpha^{2s} \tag{10}$$

In view of (9) ,we have

$$x = (7k-5)(7k^2-10k+4)^2\,\alpha^{5s} \tag{11}$$

Thus (10) and (11) represent the integer solutions to (1).

Method 4:

Taking

$$x = (7k-2)\,y \tag{12}$$

in (1) , it simplifies to

$$(7k^2-4k+1)\,y^2 = z^5$$

which is satisfied by

$$y = (7k^2-4k+1)^2\,\alpha^{5s}\,,\, z = (7k^2-4k+1)\,\alpha^{2s} \tag{13}$$

In view of (12) ,we have

$$x = (7k-2)(7k^2-4k+1)^2\,\alpha^{5s} \tag{14}$$

Thus (13) and (14) represent the integer solutions to (1).

Method 5:

The substitution

$$y = 2nz^2 \tag{15}$$

in (1) leads to

$$x^2 = z^4(7z-12n^2) \tag{16}$$

After some algebra ,it is seen that (16) is satisfied by

$$z = (7k^2-8k+4)n^2\;, x = (7k-4)(7k^2-8k+4)^2\,n^5 \tag{17}$$

In view of (15) ,one has

$$y = 2(7k^2-8k+4)^2\,n^5 \tag{18}$$

Thus,(17) and (18) satisfy (1).

Note 4:

It is to be noted that (16) is also satisfied by

$$z=(7k^2-6k+3)n^2\ ,x=(7k-3)(7k^2-6k+3)^2n^5 \tag{19}$$

In view of (15) ,one has

$$y=2(7k^2-6k+3)^2n^5 \tag{20}$$

Thus,(19) and (20) satisfy (1).

Method 6:

The substitution

$$y=(2n-1)z^2 \tag{21}$$

in (1) leads to

$$x^2=z^4(7z-3\,(2n-1)^2) \tag{22}$$

After some algebra ,it is seen that (22) is satisfied by

$$z=(7k^2-4k+1)(2n-1)^2\ ,x=(7k-2)(7k^2-4k+1)^2(2n-1)^5 \tag{23}$$

In view of (21) ,one has

$$y=(7k^2-4k+1)^2(2n-1)^5 \tag{24}$$

Thus,(23) and (24) satisfy (1).

Note 5:

It is to be noted that (22) is also satisfied by

$$z=(7k^2-10k+4)(2n-1)^2\ ,x=(7k-5)(7k^2-10k+4)^2(2n-1)^5 \tag{25}$$

In view of (21) ,one has

$$y=(7k^2-10k+4)^2(2n-1)^5 \tag{26}$$

Thus,(25) and (26) satisfy (1).

Remark : In a similar manner,one may consider the relations between x,z as in (15) ,(21) and obtain the corresponding four more sets of integer solutions to (1).

Observation: Generation formula

Let (x_0, y_0, z_0) be a particular solution to (1).

Then ,the formula for generating a sequence of integer solutions to (1) is presented below:

$$x_s = \frac{\alpha^s + 3\beta^s}{4} x_0 + \frac{3(\alpha^s - \beta^s)}{4} y_0,$$
$$y_s = \frac{\alpha^s - \beta^s}{4} x_0 + \frac{3\alpha^s + \beta^s}{4} y_0,$$
$$z_s = 2^{2s} z_0 \quad, s = 1,2,3,\ldots$$

where

$$\alpha = 2^5 ,\beta = -2^5$$

An example has been given in Table 2 below:

Table 2- Example

s	x_s	y_s	z_s
0	2	1	1
1	2^4	$3*2^4$	2^2
2	2^{11}	2^{10}	2^4
3	2^{14}	$3*2^{14}$	2^6

Conclusion:

In this paper ,an attempt has been made to obtain non-zero distinct integer solutions to the ternary non-homogeneous quintic equation $x^2 + 3y^2 = 7z^5$

As the quintic equations are rich in variety, one may search for the integer solutions to other choices of quintic equations with three or more unknowns.

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